Circles: Recap!

Highlights from the previous week:

- Circumference and area

  $C = 2\pi r$ or πd

  $A = \pi r^2$

- Arc measure and length

  Arc measure = central angle

  Arc length = $\frac{m\overset{\frown}{AB}}{360} * 2\pi r$

  circle fraction, circumference

- Sector and segment area

  Sector = $\frac{m\overset{\frown}{AB}}{360} * \pi r^2$

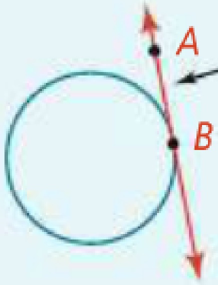
  Segment = sector − triangle

Chapter 12 - Circles

Lesson objectives: to expand our knowledge of the properties of circles to include inscribed and circumscribed angles and coordinate geometry and to use those properties to solve problems.

Tangent Lines

Tangent lines are lines that intersect a circle at exactly one point.



A **tangent to a circle** is a line in the plane of the circle that intersects the circle in exactly one point.

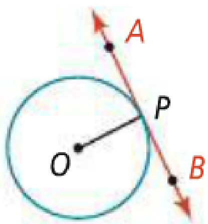
The point where a circle and a tangent intersect is the **point of tangency**.

$\overrightarrow{BA}$ is a tangent ray and $\overline{BA}$ is a tangent segment.

Theorem 12-1

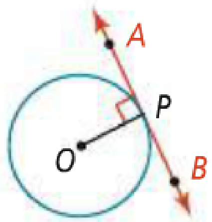
If . . .

$\overleftrightarrow{AB}$ is tangent to $\odot O$ at P



Then . . .

$\overleftrightarrow{AB} \perp \overline{OP}$

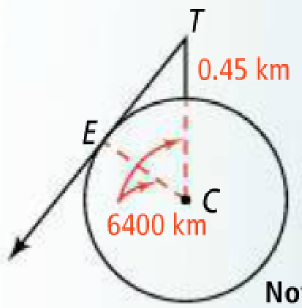


If a line is tangent to a circle, then the line is perpendicular to a radius at the point of tangency.

Example Problem

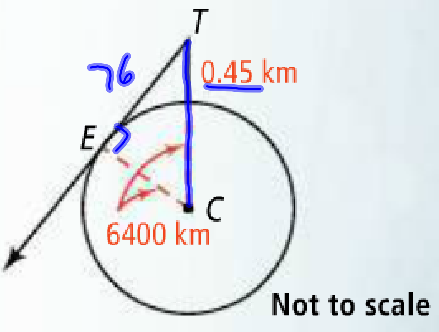
Earth Science The CN Tower in Toronto, Canada, has an observation deck 447 m above ground level. About how far is it from the observation deck to the horizon? Earth's radius is about 6400 km.

Step 1 Make a sketch. The length 447 m is about 0.45 km.



Example Problem

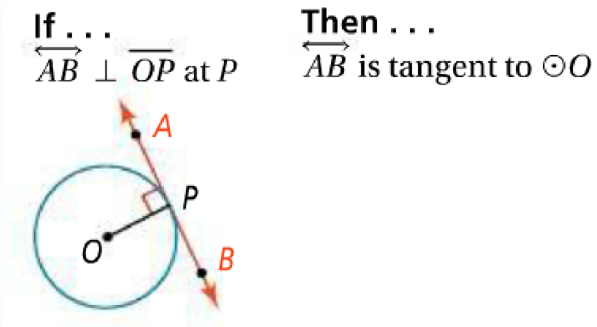
Step 2 Use the Pythagorean Theorem.



$$CT^2 = TE^2 + CE^2$$
$$(6400 + 0.45)^2 = TE^2 + 6400^2$$
$$(6400.45)^2 = TE^2 + 6400^2$$
$$40{,}965{,}760.2025 = TE^2 + 40{,}960{,}000$$
$$5760.2025 = TE^2$$
$$76 \approx TE$$

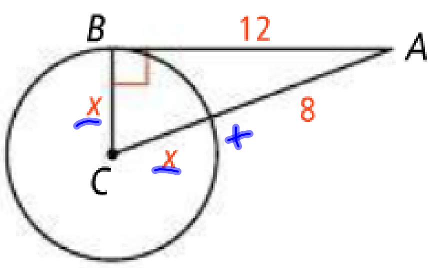
$$\underbrace{hyp}_{\text{longest side}}{}^2 = leg_1^2 + leg_2^2$$

Theorem 12-2



If a line in the plane of a circle is perpendicular to a radius at its endpoint, then the line is tangent to the circle at that point.

Example - finding a radius



What is the radius of $\odot C$?

$$AC^2 = AB^2 + BC^2$$

$$(x + 8)^2 = 12^2 + x^2$$

$$x^2 + 16x + 64 = 144 + x^2$$

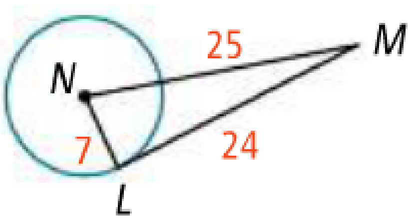
$$-x^2 \qquad\qquad -x^2$$

$$16x = 80$$

$$x = 5$$

$\overline{AB}$ is tangent to $\odot C$; $x=5$

Example - finding a tanget



Is $\overline{ML}$ tangent to $\odot N$ at L? Explain.

$$NL^2 + ML^2 \stackrel{?}{=} NM^2$$

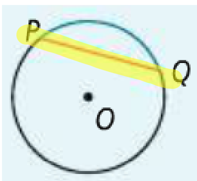
$$7^2 + 24^2 \stackrel{?}{=} 25^2$$

$$625 = 625$$

Segment ML is tangent to circle N at point L because it is perpendicular to the radius at that point.

Chords

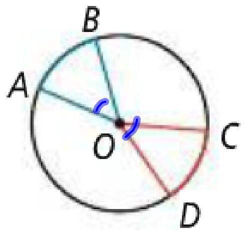
Chords are line segments that do not pass through the center of the circle and whose endpoints are on the circumference of the circle.



Segment PQ is a chord in circle O.

Theorem 12-4

Within a circle or in congruent circles, congruent central angles have congruent arcs.

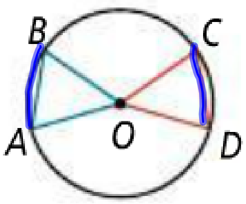


If $\angle AOB \cong \angle COD$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.
If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\angle AOB \cong \angle COD$.

Converse:
Similarly, congruent arcs have congruent central angles.

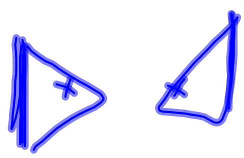
Theorem 12-5

Within a circle or in congruent circles, congruent central angles have congruent chords.



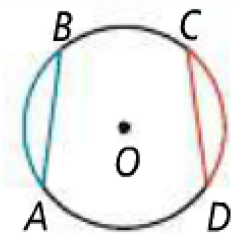
If $\angle AOB \cong \angle COD$, then $\overline{AB} \cong \overline{CD}$.
If $\overline{AB} \cong \overline{CD}$, then $\angle AOB \cong \angle COD$.

Converse:
Similarly, congruent chords have congruent central angles.



Theorem 12-6

Within a circle or in congruent circles, congruent chords have congruent arcs.



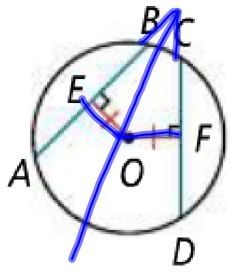
If $\overline{AB} \cong \overline{CD}$, then $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

If $\overset{\frown}{AB} \cong \overset{\frown}{CD}$, then $\overline{AB} \cong \overline{CD}$.

Converse:
Similarly, congruent arcs have congruent chords.

Theorem 12-7

Within a circle or in congruent circles, chords equidistant from the center(s) are congruent.

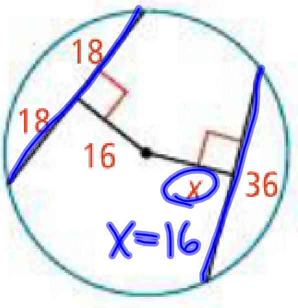


If $OE = OF$, then $\overline{AB} \cong \overline{CD}$.

If $\overline{AB} \cong \overline{CD}$, then $OE = OF$.

Converse:
Similarly, congruent chords are equidistant from the center(s).

Example 12-7

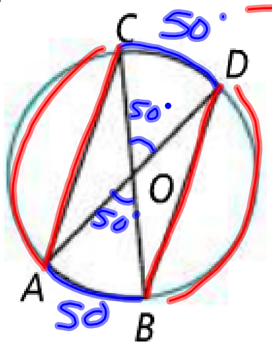


What is the value of x? Explain.

Converse of 12-7: congruent chords are equidistant from the center(s).

Another Example

In ⊙O, $m\overset{\frown}{CD} = 50$ and $\overline{CA} \cong \overline{BD}$.



1. What is $m\overset{\frown}{AB}$? How do you know?

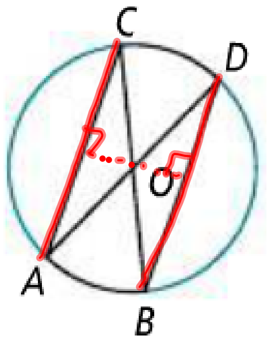
50; vertical angles; equal central angles have equal arcs so $m\overset{\frown}{AB}$ = $m\overset{\frown}{CD}$.

2. What is true of $\overset{\frown}{CA}$ and $\overset{\frown}{BD}$?

$\overset{\frown}{CA}$ and $\overset{\frown}{BD}$ are congruent because congruent chords have congruent arcs.

Another Example (continued)

In $\odot O$, $m\overset{\frown}{CD} = 50$ and $\overline{CA} \cong \overline{BD}$.



3. Since $\overline{CA}=\overline{BD}$ what is the distance of $\overline{CA}$ and $\overline{BD}$ from the center of circle O?

Equal since congruent chords are equidistant from the center.

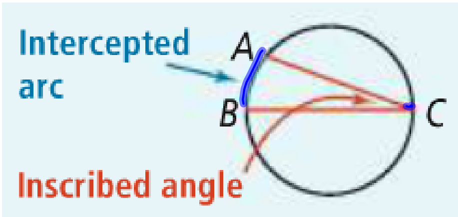
Vocabulary

Vocabulary Is a radius a chord? Is a diameter a chord? Explain your answers.

A radius is NOT a chord because both endpoints of a radius are not on the circumference.

A diameter is NOT a chord because a diameter goes through the center of a circle.

Inscribed Angles



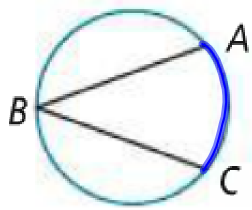
What is an "inscribed" angle?

An angle whose vertex is on the circle and whose sides are chords of the circle is an **inscribed angle.** An arc with endpoints on the sides of an inscribed angle, and its other points in the interior of the angle is an **intercepted arc.** In the diagram, inscribed $\angle C$ intercepts $\widehat{AB}$.

Theorem 12-11

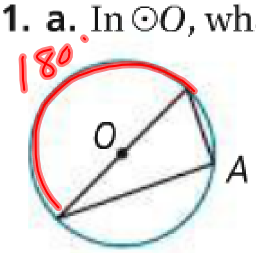
The measure of an inscribed angle is half the measure of its intercepted arc.

$$m\angle B = \frac{1}{2} m\widehat{AC}$$



$m\widehat{AC} = 50°$
$\angle B = \frac{1}{2} * 50°$
$= 25°$

1. a. In $\odot O$, what is $m\angle A$?



90 degrees

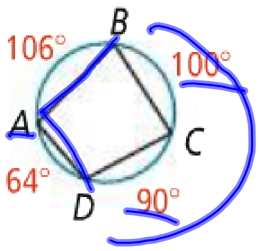
$m\angle A = \frac{1}{2}(180°)$
$= 90°$

Example

The measure of an inscribed angle is half the measure of its intercepted arc.

$$m\angle B = \frac{1}{2} m\widehat{AC}$$

b. What are $m\angle A$, $m\angle B$, $m\angle C$, and $m\angle D$?

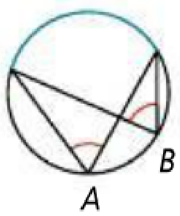


$m\angle A = \frac{100° + 90°}{2}$

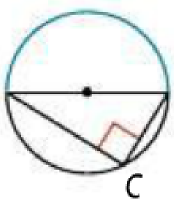
A: 95
B: 77
C: 85
D: 103

Corollaries to theorem 12-11

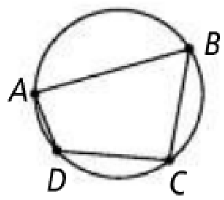
Corollary 1
Two inscribed angles that intercept the same arc are congruent.



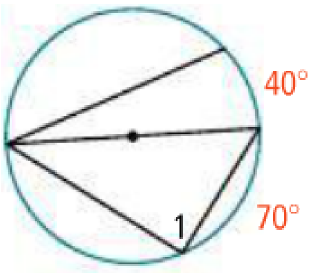
Corollary 2
An angle inscribed in a semicircle is a right angle.



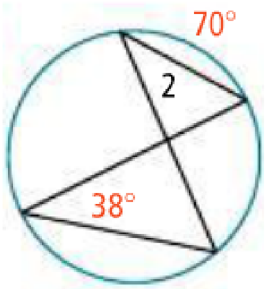
Corollary 3
The opposite angles of a quadrilateral inscribed in a circle are supplementary.



Examples



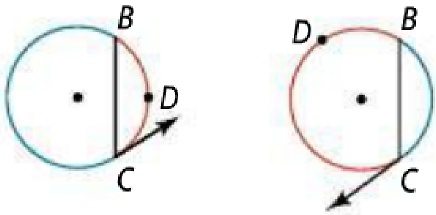
$\angle 1$ is inscribed in a semicircle. By Corollary 2, $\angle 1$ is a right angle, so $m\angle 1 = 90$.



$\angle 2$ and the 38° angle intercept the same arc. By Corollary 1, the angles are congruent, so $m\angle 2 = 38$.

Theorem 12-12

The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.



$$m\angle C = \frac{1}{2} m\widehat{BDC}$$